EURO J Decis Process
DOI 10.1007/s40070-014-0038-5

ORIGINAL ARTICLE

Subjectively biased objective functions

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Received: 29 December 2013 / Accepted: 10 December 2014

Abstract The maximization of an objective function is a cornerstone of OR/MS modeling. How can we integrate subjective values within these models without weakening their scientific objectivity? This paper proposes a methodological answer that maintains the objective function and relaxes the maximization principle. We introduce a class of biased models that combine an objective function with a "subjective" factor that biases the maximization of such a function. We present the main properties of these models as well as the axiomatic foundations that allow for the rigorous measurement of biasing factors. We invite OR/MS scholars to participate in the development of practical applications integrating ethical and sustainability values.

Keywords Modeling · Maximization principle · Ethics · Sustainability · Bias · Threshold · Measurement error · Sustainable procurement · Precautionary principle · Renewable electricity

1 Introduction

The maximization of an objective function is a cornerstone of OR/MS modeling. How can we integrate subjective values within these models without weakening their scientific objectivity? This paper proposes a methodological answer that maintains the objective function and relaxes the maximization principle. We

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Published online: 09 January 2015

introduce a class of biased models that combine an objective function with a "subjective" factor that biases the maximization of such a function[1]. Building on papers and results obtained in the last ten years, we want to open a debate about the treatment of subjective values within OR/MS modeling.

In previous work (Le Menestrel and Van Wassenhove 2004), we argue that integrating subjective values, and especially ethical or sustainability values, within OR/MS maximization models bears the risk of weakening the scientific rigor and legitimacy of these models. Theoretically, this argument holds in the sense that the measurement of an alternative is as good as the weakest of its dimensions. If we include within the objective function values that are poorly defined, too dependent on context or of essentially interpretive nature, we lose objectivity of the whole function. Practically, including too many subjective dimensions in the quantitative evaluation opens space for abusive interpretations that would be criticized by stakeholders and/or by decision makers. More generally, such considerations have led the OR/MS community to treat subjective values, and in particular ethical values, outside of their models and with a sense of disregard (Le Menestrel and Van Wassenhove 2008).

On the other hand, there are strong arguments in favor of OR/MS models to better integrate subjective values such as ethical values, sustainability considerations, cultural tastes or even spiritual concerns (Ackoff 1956; Churchman 1970; Gass 1991, 1994; Wallace 1994; Ormerod 1996; Brans and Gallo 2007; Ormerod and Ulrich 2013 for a recent literature review). Fairness considerations, negative externalities, procedural concerns, values for a precautionary approach towards unknown consequences of technology, principled-based commitments to sustainable development, respect of professional codes are all examples of good reasons to depart from a purely objective evaluation. It is undeniable that OR/MS analysts are increasingly confronted with complex social problems for which a modeling that excludes these considerations limits the relevance of their work. Moreover, because OR/MS analysts have some knowledge of the considerations they exclude to be objective in their evaluation, they could use that knowledge to propose a more inclusive help to decision makers. As demonstrated in a recent series of Special Issues (e.g., Le Menestrel and Van Wassenhove 2009), approaches that embed ethical values in OR/MS models and position them as aid to decision making increasingly encounter interest of analysts (Roy 1993; Brans 2002 are seminal references in this respect).

In this paper, we propose an innovative approach towards the trade-off between scientific objectivity on the one hand and subjective values on the other hand. This approach models the combination of two separate types of values that influence preferences. Firstly, there is the value that we can attribute to the alternatives we choose and that depends on their observable quantities. These values are measured by an objective function whose characteristics are objective enough to make it fully quantitative, i.e., a ratio-scale. Secondly, there are subjective considerations that

[1] Contrasting objectivity and subjectivity, we play here with the two meanings of "objective": what the decision maker wants to achieve (the goal) and scientific objectivity. The reader is asked to patiently wait for a more precise discussion of what these terms mean in the present paper.

may also influence preferences but that we cannot objectively express as a quantity of something directly observable. These values can only be revealed by observing preferences and are measured by a multiplicative factor outside of the objective function. Preferences, in general, may depart from the maximization of the objective function (see Sen 1997 for a convincing argument against the maximization principle in human rationality).

In this approach, the meaning of this subjective and behaviorally revealed factor is not directly modeled. It depends on the interpretation of the model, interpretation that may be different for the decision maker or for the modeler. In some cases, it may be interpreted as a value for fairness, for reciprocity, or for honesty. In some others, it may be interpreted as the willingness to respect a rule or a principle, or a consideration for consequences externalized by the objective function. The factor may also reflect an emotional bias towards a specific alternative over another. Methodologically, the interpretation of subjective values is not part of the mathematical design but left out for discursive analysis taking place outside of the modeling process (Rauschmayer et al. 2009). Only the empirically observed influence of these subjective factors on the model is mathematically treated through the biasing function.

The innovative methodological proposal of treating subjective values outside the objective function fundamentally differs from including these values within the objective function. Mathematically, the range of situations that can be modeled is broader than in maximization models. We do not need to assume that indifference of preferences is an equivalent relation and can model situations where interval orders, for instance, are more appropriate models for the type of ordering empirically observed (Fishburn 1997; Öztürk 2008; Kronus 2011; Öztürk et al. 2011 for theoretical discussions). Federgruen et al. (2007), Gallego et al. (1992), Inuiguchi and Mizoshita (2011), Jansen (1993), Jianzhong and Dan (1994), Kim et al. (2011) for applications. We can even model situations where subjective considerations induce violations of transitivity, incomplete preferences or preference reversal (Fishburn 1991). Practically, the objective function remains a convenient object to measure and interpret the quantitative values of alternatives. In some cases, it may simply be the price of each alternative, as a function of the different quantities that compose this alternative. Subjective values then appear as a factor that distorts such a price and this factor models a proportional premium or discount expressed as a percentage.

The introduction of a new class of models always raises communication difficulties, in particular because the new approach competes with well-established ones. Since our first exploration of models where subjective imprecision would be treated outside of the functional measurement of alternatives (Le Menestrel and Van Wassenhove 2001), we have focused our efforts on establishing mathematical foundations that clarify the measurement-theoretic specificities of these models. We now want to invite scholars to explore practical applications of biased models and open a debate about subjective values and OR/MS models.

The rest of the paper is structured as follows. In Sect. 2, we present the models and their main constituents: the objective function and the subjective biasing factor. In Sect. 3, we present some main properties of these models, making more explicit how the biasing factor captures a type of considerations that is usually not included in maximization models. In Sect. 4, we present the key axioms which support their measurement-theoretic foundations. In Sect. 5, we sketch typical applications that we have in mind for biased models. A brief Sect. 6 concludes.

2 Presentation of biased models

Instead of integrating subjective values within the objective function, we reflect these values through a biasing function which skews the objective function and distorts the maximization process. Let x and y be elements of a set of alternatives. Let f be an objective function that measures the value of these alternatives. Let α be a biasing function. Then,

an alternative x is preferred to another alternative y

if and only if

$$\alpha(x,y)f(x) \geq f(y).$$

2.1 The Objective function

As a linear ratio-scale, the objective function provides for an absolute, quantitative and objective measure of the value of alternatives.

Ratio-Scale In biased models, the objective function is unique up to multiplication by a positive scalar. As such, it has a unit (up to multiplication by a positive scalar) and specifies an origin, which is naturally interpreted as the absence of alternative.

Positive linear quantitative measurement the objective function varies positively and linearly with the quantity of a particular alternative. It verifies the property $f(\lambda x) = \lambda f(x)$, with λ being a positive scalar. This is the property that allows for the uniqueness of the measurement of the biasing function.

Absolute measurement as a ratio-scale that associates a number to each alternative individually, the objective function measures the value of each alternative independently of other alternatives.

Objective measurement Since the function is a ratio-scale, it allows meaningful ratio comparisons and statistics, even across decision makers (Stevens 1946). In this sense, the objective function measures the value of an alternative independently of subjective considerations.

2.2 The biasing function

The biasing function captures the extent to which preferences depart from the maximization of the objective function. It is a factor reflecting the “quality” of each alternative, independently of its quantity and which biases the preferred alternative relative to another. It is unique and depends on each subject who expresses preferences and characterizes its specificity.

Unique factor the biasing function is uniquely determined by the preferences and the objective function. Without dimension, it has no unit and no origin is naturally specified. It is naturally interpreted as a factor.

Qualitative nature the biasing function does not vary with the quantity of alternatives. It verifies the property $\alpha(\lambda x, \lambda y) = \alpha(x, y)$, with λ being a positive scalar.

Relative measurement the biasing function is a function of the two alternatives over which preferences are expressed. This multiplicative factor potentially varies if one of the alternatives changes while the other is kept constant.

Subjective measurement if all preferences of this model share the same objective function, the biasing function characterizes a particular relation of preferences. Since these preferences are the expression of a decision maker, the biasing function characterizes the specificity of a particular decision maker. In this sense, it is subjective.

3 Properties of biased models

In this model, except if the biasing function is equal to 1 (in which case, we fall back to a standard maximization model) it is not necessary nor sufficient that an alternative has more value than another to be preferred. In this manner, a relative bias specific to a particular decision maker explains the departure from the principle of maximization. This allows the modeling of a variety of properties that the maximization of an objective function must exclude.

Proportional threshold When the biasing factor is strictly smaller than 1, there are pairs of alternatives for which none is preferred to the other. Alternatives cannot be discriminated against one another if the ratio of their objective values is below that factor. This indifference takes place among alternatives which are not equivalent and it is not transitive, a phenomenon well known to scholars investigating measurement empirically. When the biasing factor is strictly greater than 1, there are pairs of alternatives for which each alternative is preferred to the other. This reflects a form of imprecision or indecisiveness according to which it is not possible to predict which alternative will be chosen by the decision maker. These properties are reflected in Fig. 1.

Thick indifference curves Another way to express the influence of the biasing factor is through indifference curves. Subjective values are not modeled as leading to a trade-offs with objective values similar as the trade-offs between dimensions of the objective function. If indifference curves represent combinations of different dimensions of an alternative that are indifferent one with the other, the biasing factor creates a thickness of these indifference curves. In Fig. 2, we take the example of alternatives that are composed of quantities of two dimensions x_1 and x_2. The dotted line indicates the combinations that have the same valuation with the objective function. Plain lines delineate the set of indifferent combinations due to a biasing factor which is, in this example, constant.

Error in measurement Biased models reflect empirical situations where specific characteristics of the decision maker in a particular situation combine with

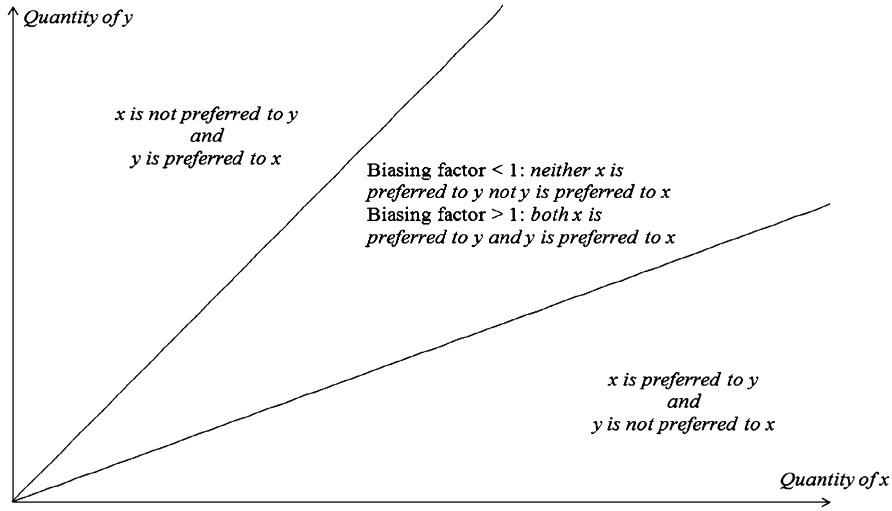

Fig. 1 A proportional subjective threshold of indecisiveness

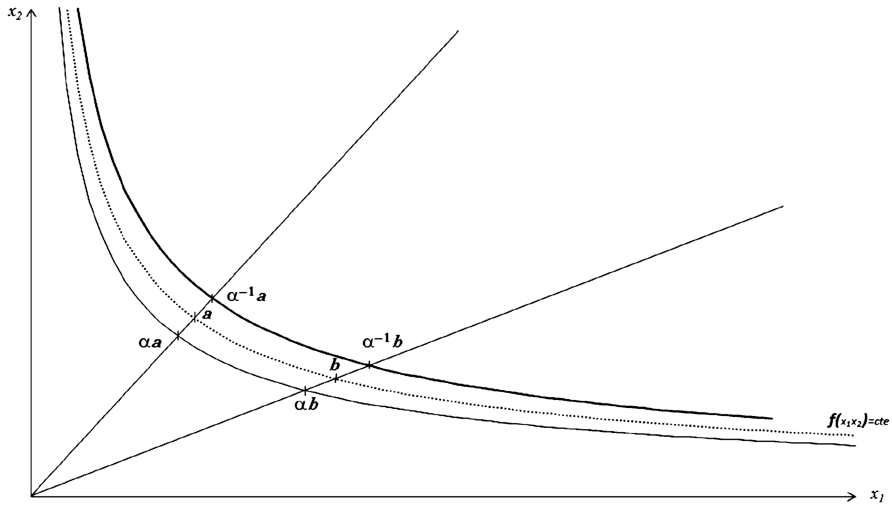

Fig. 2 Subjective values as inducing thickness of indifference curves

properties of the alternatives to form preferences. As such, they reflect a variability of empirical observation typically interpreted as measurement error. In biased models, it is however not a lack of precision in the measurement of alternatives that induces the variability of preferences. Both alternatives and the biasing function are precisely measured. The lack of discrimination and the indecisiveness of the models stem from the combination of the ratio-scale and the biasing factor.

The value of subjective values It is interesting to note that biased models do not assign subjective considerations a specific value as measured by the objective function. Measured with a multiplicative factor that has no dimension, subjective values have a proportional influence on objective values. Therefore, the impact of subjective values on objective values varies with the latter.

4 Axioms for biased models

The conditions for the existence and uniqueness of both an objective function and a biasing function have been studied as an extension of the theory of representational measurement (Krantz et al. 1971). A series of mathematical results have been obtained in progressively more and more general settings (see Le Menestrel and Lemaire 2004 for a first homogeneous setting).The main features of such representation theorems are as follows:

Replicative structures Given any particular alternative, it is supposed that we can construct another alternative by replicating it. Mathematically, let A denote a non-empty set of objects x, y, z and $\mathbb{N}^*$ the set of positive integers. The structure of alternatives consists of the set A together with a map $\mathbb{N}^* \times A \rightarrow A, (m, x) \longmapsto mx$ such that $(mm')x = m(m'x)$ and $1x = x$. Such a structure is called a $\mathbb{N}^* - set$. It is more general than the semi-group structures which are referred to in the classical theory of representational measurement. Moreover, these structures are general enough for the results to hold for positive cones of real vector spaces of any dimension L, i.e., for $A = (\mathbb{R}_+^*)^L$. It is with such a structure in mind that we have composed the examples of this paper and naturally interpreted λx as the quantity λ ($\lambda \in \mathbb{R}_+^*$) of an alternative x ($x \in A$), which can itself be composed of several dimensions $x_1, x_2, \ldots, x_L$. In practice, this means that only properties or attributes for which a replication makes operationally sense enter into the domain of the objective function. In the sustainable procurement example below, this leads to naturally consider items to be bought as composing that domain, leaving to the biasing function the subjective appreciation of their sustainability.

Homothetic structures From the measurement-theoretic point of view, a most important property necessary for such model to hold is the one of homotheticities. Mathematically, if $\succ$ is a preference relation over A, then $\succ$ is homothetic if and only if it verifies the property $x \succ y \Leftrightarrow \lambda x \succ \lambda y$ ($\lambda \in \mathbb{R}_+^*$). Practically, this property requires that preferences are scale-invariant: they do not change when all quantities are multiplied by a positive scalar. For instance, preferences between procuring an item x or an item y, both meeting the desired specifications but differing in their sustainability character, would not empirically vary if the choice consists of buying one unit of each of these items or two units, three units, etc.

Positive structures It is supposed that alternatives are positive in the sense that more of a given alternative is desirable. Mathematically, $x \succ y \Rightarrow \lambda x \succ y$ ($\lambda \in \mathbb{R}_+^*$). Practically, if an alternative is preferred to another, then more of this alternative is also preferred, which we consider a rather uncontroversial axiom to consider.

Ordered structures Various results have been obtained for preference relations which are not necessarily complete nor transitive. In particular, it has been shown that positive homothetic structures endowed with a semiorder can be represented by a ratio-scale and a constant bias smaller than 1 (Le Menestrel and Lemaire 2006a; Lemaire and Le Menestrel 2006). When they are endowed with an interval order, this bias remains smaller than 1 but is not necessarily constant. A more general result has been obtained for positive homothetic structures endowed with a biorder (also called a Ferrer relation in that case, Doignon et al. 1984). In that more general case, the biasing function may be greater than 1, thus allowing preferences for alternatives that have lower objective values (Le Menestrel and Lemaire (2006b); Lemaire, and Le Menestrel (2009)). Mathematically, if $\succ$ is a preference relation over A, then $\succ$ is a biorder if and only if it verifies the property $(x \succ y$ and $z \succ t) \Rightarrow (x \succ t$ or $z \succ y)$.

Other technical conditions Depending on the representation theorem, separability and Archimedean conditions complement the main properties above.

5 Sketching a few examples for applications

5.1 A premium for sustainable procurement

Large corporations use auctions to make purchases of thousands of different items from suppliers all over the world. These auctions used to be price only, i.e., the lowest bidder would get the deal. In recent times, other criteria have become important for the buyer (Beil and Wein 2003). One obvious example is quality. Most of the time, this attribute can be measured and therefore it can be used as a qualifier, i.e., every supplier who does not reach the minimum required quality cannot participate in the auction (Branco 1997). However, recently we have seen that some attributes that are less quantifiable become more important. One such example is sustainability. Some suppliers are clearly more sustainable in their practices than others but this attribute may be difficult to measure and its appreciation may be subjective. One can therefore imagine an auction where the price would be adjusted by a factor that reflects sustainability of the supplier.

5.2 A precautionary discount in case of ignorance of consequences

When experts acknowledge uncertainty, they tend to do so in ways that reduce unknowns to measurable ‘risk’. This leaves science advice vulnerable to the social dynamics of groups—and to manipulation by political pressures seeking legitimacy, justification and blame management (Stirling 2010, p. 1029). Although we cannot directly measure the unpredictable, there are practical processes by which decision makers’ and stakeholders’ subjective appreciation of uncertainty or ignorance of consequences can be revealed and usefully integrated in the analysis (Springborn et al. 2013). Research on uncertainty indeed suggests that individuals apply a discount when they do not know the probability of outcomes. Ignorance of consequences, a stronger notion than uncertainty, may also be appropriately reflected by a bias to be measured. It would be particularly interesting to systematically measure such values as a bias, studying how the intensity of these subjective considerations varies with groups and information.

5.3 Constant premium over renewable electricity sources

Consider preferences between kWh of electricity produced using non-renewable resources and kWh of electricity produced using renewable resources. Such preferences are likely to combine an objective component with some subjective component (Kotchen and Moore 2007; Ek and Söderholm 2008; van den Bergh 2008). Economic factors are captured by an objective function $f(.)$ that associates a numerical valuation to energy given a series of assumptions and parameters. A biased model also considers a biasing function, measured by observing preferences between renewable and non-renewable sources. With a particular decision maker, we may observe a maximum premium of, for instance 25 % for renewable over non-renewable. This would correspond to the semiorder model:

$$\text{A quantity } x \text{ of renewable electricity is preferred to a quantity } y$$
$$\text{of non}-\text{renewable electricity if and only if } \frac{5}{4} f(x) \geq f(y).$$

5.4 Combinations of electricity sources and variable bias

This model can be extended to a mix of energy sources. In that case, alternatives consist of bundles of quantity of renewable and non-renewable electricity. Let λx_1 denote the quantity λ of renewable energy x_1 and μx_2 denote the quantity μ of non-renewable energy x_2. For instance, consider the biasing function $\alpha(x, y) = \gamma(x)\gamma(y)$ with $\gamma(x) = \frac{\lambda x_1 + \mu x_2}{x_1 + x_2}$. The semiorder model above would be a special case of the more general interval order model:

$$\text{A bundle } x = \lambda x_1 + \mu x_2 \text{ of renewable and non}-\text{renewable electricity}$$
$$\text{is preferred to a bundle} y \text{ if and only if} \gamma(x)\gamma(y) f(x) \geq f(y),$$
$$\text{with} \gamma(x) = \frac{\lambda x_1 + \mu x_2}{x_1 + x_2}.$$

In that case, we would have $\lambda\mu = \frac{5}{4}$.

5.5 Distribution of biases across decision makers

With biased models, heterogeneity of decision makers could be reflected through distribution of biases. For instance, it may happen that, in a given population, a maximum bias of 25 % is an average. The study of such distribution could be of interest to reflect the variability of subjective values within a population.

6 Further research and concluding remarks

We see at least three main avenues for further research around the approach of subjectively biased objective functions. Firstly, the empirical robustness of the structural axioms necessary for biased measurement (especially the homotheticity invariance property) should be studied. Secondly, methods to elicit the biasing function should be investigated and compared with the ones that have been developed to elicit utility functions. Thirdly, practical examples should be more fully developed so as to clarify further the distinctive contribution of treating subjective considerations outside the objective function. We hope this paper will inspire the OR/MS community to investigate these questions, empowering OR/MS models to consider subjective considerations in a rigorous and scientific manner.

Acknowledgments Marc Le Menestrel acknowledges financial support by Spanish Ministry of Economy and Competitiveness grants MINECO-ECO2011-25607 and MINECO-ECO2011-25295.

References

Ackoff RL (1956) The development of operational research as a science. Oper Res 4:265–295

Beil DR, Wein LM (2003) An inverse-optimization-based auction mechanism to support a multi-attribute RFQ process. Manag Sci 49(11):1529–1545

Branco F (1997) The design of multidimensional auctions. RAND J Econ 28(1):63–81

Brans JP (2002) Ethics and decision. Eur J Oper Res 136:340–352

Brans J-P, Gallo G (2007) Ethics in OR/MS: past, present and future. Ann Oper Res 153(1):165–178

Churchman CW (1970) Operations research as a profession. Manag Sci 17(2):B37–B53

Doignon JP, Ducamp A, Falmagne JC (1984) On realizable biorders and the biorder dimension of a relation. J Math Psychol 28:73–109

Ek K, Söderholm P (2008) Norms and economic motivation in the Swedish green electricity market. Ecol Econ 68:169–182

Federgruen A, Meissner J, Tzur M (2007) Progressive interval heuristics for multi-item capacitated lot-sizing problems. Oper Res 55(3):490–502

Fishburn PC (1991) Nontransitive preferences in decision theory. J Risk Uncertain 4:113–134

Fishburn PC (1997) Generalizations of semiorders: a review note. J Math Psychol 41:357–366

Gallego G, Shaw D, Simchi-Levi D (1992) The complexity of the staggering problem, and other classical inventory problems. Oper Res Lett 12(1):47–52

Gass SI (1991a) The many faces of OR. J Oper Res Soc 42:3–15

Gass SI (1994) On ethics in operational research. J Oper Res Soc 45:965–966

Inuiguchi M, Mizoshita F (2011) Qualitative and quantitative data envelopment analysis with interval data. Ann Oper Res 1–32

Jansen K (1993) Scheduling with constrained processor allocation for interval orders. Comput Oper Res 20(6):587–595

Jianzhong Z, Dan S (1994) A priority measure in interval methods for constrained global optimization. Comput Oper Res 21(5):511–519

Kim H, Huo X, Shi J (2011) A single interval based classifier. Ann Oper Res 1–19

Kotchen MJ, Moore MR (2007) Private provision of environmental public goods: household participation in green-electricity programs. J Environ Econ Manag 53:1–16

Krantz DH, Luce RD, Suppes P (1971) Foundations of measurement, vol 1. Academic press, New York

Kronus D (2011) Relations between threshold and k-interval Boolean functions. Ann Oper Res 188(1):263–278

Lemaire B, Le Menestrel M (2006) Homothetic interval orders. Discrete Math 306:1669–1683

Lemaire B, Le Menestrel M (2009) Generalized homothetic biorders. Discrete Math 309:3793–3810

Le Menestrel M, Lemaire B (2004) Biased extensive measurement: the homogeneous case. J Math Psychol 48:9–14

Le Menestrel M, Lemaire B (2006a) Biased extensive measurement: the general case. J Math Psychol 50:570–581

Le Menestrel M, Lemaire B (2006b) Ratio-scale measurement with intransitivity or incompleteness: the homogeneous case. Theory Decis 60:207–217

Le Menestrel M, Van Wassenhove LN (2001) The domain and interpretation of utility functions: an exploration. Theory Decis 51(2–4):329–349

Le Menestrel M, Van Wassenhove LN (2004) Ethics outside, within or beyond OR models? Eur J Oper Res 153:477–484

Le Menestrel M, Van Wassenhove LN (2008) "7 Barriers to Ethics in OR/MS" Ifors newsletter (March)

Le Menestrel M, Van Wassenhove LN (2009) Ethics in operations research and management sciences: a never-ending effort to combine rigor and passion. Omega 37:1039–1043

Ormerod RJ (1996) On the nature of OR: entering the fray. J Oper Res Soc 47:1–17

Ormerod RJ, Ulrich W (2013) Operational research and ethics: a literature review. Eur J Oper Res 228(2):291–307

Öztürk M (2008) Ordered sets with interval representation and (m, n)-Ferrers relation. Ann Oper Res 163(1):177–196

Öztürk M, Pirlot M, Tsoukiàs A (2011) Representing preferences using intervals. Artif Intell 175(7–8):1194–1222

Rauschmayer F, Kavathatzopoulos I, Kunsch PL, Le Menestrel M (2009) Why good practice of OR is not enough–ethical challenges for the OR practitioner. Omega 37:1089–1099

Roy B (1993) Decision science or decision-aid science? Eur J Oper Res 66:184–203

Sen A (1997) Maximization and the act of choice. Econometrica 65:745–779

Springborn M, Yeo BL, Lee J, Six J (2013) Crediting uncertain ecosystem services in a market. J Environ Econ Manag 66(3):554–572

Stevens S (1946) On the theory of scales of measurement. Science 103:122–136

Stirling A (2010) Keep it complex. Nature 468:1029–1031

van den Bergh JCJM (2008) Environmental regulation of households: an empirical review of economic and psychological factors. Ecol Econ 66:559–574

Wallace WA (ed) (1994) Ethics in modeling. Elsevier Science Ltd., Oxford